

BOOSTER VACUUM CHAMBER CONSIDERATIONS

*Booster Technical Note*  
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The AGS Injection Booster is a rapid cycling accelerator when used to accelerate protons. This rapid change in the magnetic field induces currents in the metalic vacuum chamber which in turn creates unwanted harmonics in the field shape. To reduce these magnetic field distortions it is desireable to make the vacuum chamber wall as thin as possible consistent with mechanical considerations.

This note will analyze the mechanical properties of two types of possible vacuum chamber walls, solid and convoluted (corrugate along the z axis of the chamber).

SOLID

For the purposes of analysis five cross-sectional shapes will be considered.

1. Elipical, 1.5 x 4 inches
2. Elipical, 1.375 x 4.375
3. A two arc design with radii equal to 12.1875 and 1.0317 inches, see figure 1.
4. A three arc design with radii equal to 12.1875, 0.6183 and 3.0 inches, also see figure 1.
5. The AGS vacuum chamber.

The maximum deflection and peak bending stress in shapes of this kind can be calculated using the formulas given in references 1 through 4. The deflection calculation assumes a deformed cylinder. The basic equations are as follows:

$$M = \gamma P a^2$$

where  $P$  -- applied pressure

$$\Delta D = \frac{D_o P}{P_{cr} - P} \quad D = \int_{P=0}^{P=P} \Delta D$$

$a$  -- half minor diameter

$b$  -- half major diameter

$M$  -- max., bending moment

$\gamma$  -- form factor shown in figure 2

$S$  -- stress

$c$  -- centroidal distance

$I$  -- moment of inertia per unit length

$D_o$  -- deformation from a cycle

$D$  -- deflection

$P_{cr}$  -- critical pressure

$\nu$  -- poisson's ratio

$R$  -- circumference  $\div$  by  $2\pi$

$E$  -- young's modulus

$t$  -- wall thickness

Using the above relationships the data contained in Table 1 can be generated. The most favorable design, the three arc design, will require a wall thickness of 0.0838 inches, if the peak stress is to be limited to 40 Kpsi. But a wall thickness this large will contain circulating current and generate sizable harmonic distortion in the magnetic field, see reference 5.

### CONVOLUTED WALL

To reduce the wall thickness a convoluted wall can be used to help provide the needed mechanical strength. The moment of inertia for a parabolically corrugated section is given by reference 6 as follows:

$$I = \frac{64}{105} (b_1 h_1^3 - b_2 h_2^3)$$

$$\text{where: } h_1 = \frac{H+t}{2}$$

$$h_2 = \frac{H-t}{2}$$

B = base width of the convolution

$$b_1 = \frac{B+2.6t}{4}$$

t = material thickness

$$b_2 = \frac{B-2.6t}{4}$$

Upon close examination I find that the approximations used to simplify the expressions for  $b_1$  and  $b_2$  are poor for small values of H. Better approximations occur when the following is used.

$$b_1 = \frac{B \left[ 1 + \frac{t}{H} \right]^{\frac{1}{2}}}{4} \quad \text{and} \quad b_2 = \frac{B \left[ 1 - \frac{t}{H} \right]^{\frac{1}{2}}}{4}$$

For sine wave convolutions the moment of inertia can be shown to be given

by

$$I = B \left[ \frac{t^3}{12} + \frac{tH^2}{8} \right]$$

In all cases  $c \frac{H+t}{2}$

These two relationships give similar results.

If the geometry of the chamber cross-section are determined by other considerations, then  $\gamma$  and  $\sigma$  are fixed and the maximum bending moment is determined by the expression given. The peak bending stress is determined by the  $I/c$  ratio. Using the "3 arc" cross-section as an example I have prepared figure 3 which relates  $H$  and  $t$  for a constant  $I/c$  ratio of  $1.17 \times 10^{-3}$  in $^3$ . This value of the  $I/c$  ratio was chosen to limit the peak stress to 40 Kpsi which I believe is an acceptable peak stress for both inconel and stainless steel, the two material under construction.

By examining figure 3 we can see that, if the wall thickness is made small, the total space occupied by the chamber edge ( $H+t+D$ ) becomes large. However, a reasonable reduction in the wall thickness is possible with a small increase in the space assigned to the chamber periphery. The  $H+t+D$  sum increases from a minimum value of 0.177 inches for a non-convulated chamber wall to 0.237 inches for a wall thickness of 0.030 inches and a convolution height of 0.169 inches. I believe this to be a reasonable compromise.

## References

1. Bevatron Injector II Linac to Inflector Section Stresses and Deflector in IM-2 Elliptical Vacuum Tank. Stanley Johnson, LBL, Engr. Note 7309-06MZ March 26, 1962.
2. Timoshenko and Gere, Strength of Material - Part I.
3. Timoshenko, Theory of Elastic Stability.
4. Roark & Young, Formulas for Stress and Strain.
5. G. Morgan and S. Kahn, Booster Tech Note 4, Calculation of Eddy Currents in the Beam Tube.
6. Mark's Standard Handbook for Mechanical Engr., 8th Edition.

Table I

	Ellipse 1.5x4	Ellipse 1.375x4.375	2 arc	3 arc	AGS
$a_1$ in	1.5	1.375	1.375	1.375	1.5314
$b_1$ in	4	4.375	3.7778	3.50	3.4064
$a/b$	.375	.314	.364	.393	.4496
$\gamma$	1.85	2.85	1.82	1.65	1.15
$t$ , in	.0968	.1101	.0880	.0838	.078
$I$ , in $^4$	$7.56 \times 10^{-5}$	$11.12 \times 10^{-5}$	$5.68 \times 10^{-5}$	$4.90 \times 10^{-5}$	$3.95 \times 10^{-5}$
$M$ , in-lbs	62.44	80.82	51.61	46.79	40.45
$S$ , Kpsi	40.0	40.0	40.0	40.0	40.0
$R$ , in	2.894	3.074	2.798	2.706	2.716
$P_{cr}$ , psi	331	406	274	242	209
Deflection, in.	.065	.064	.080	.085	.088
$I/c$ , in $^3$	$1.56 \times 10^{-3}$	$2.02 \times 10^{-3}$	$1.29 \times 10^{-3}$	$1.17 \times 10^{-3}$	$1.01 \times 10^{-3}$

Note -  $\nu = .28$     $P = 15$  psi    $E = 30 \times 10^6$  psi





